Subcritical dyke propagation in a host rock with temperature-dependent viscoelastic properties

Zuan Chen¹ and Z.-H. Jin²

¹Key Laboratory of the Study of Earth’s Deep Interior, Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing 100029, China. E-mail: zachen@mail.igcas.ac.cn
²Department of Mechanical Engineering, University of Maine, Orono, ME 04469, USA

SUMMARY
In this paper, we examine the effects of temperature-dependent viscoelastic properties of the host rock on the subcritical growth of a dyke from a magma chamber. A theoretical relationship between the velocity of subcritical dyke growth and dyke length is established using a perturbation solution of stress intensity factor at the dyke tip and a viscoelastic crack growth theory in which the temperature-dependent creep properties are taken into account. The temperature field around the dyke is calculated using an analytic solution. The numerical results for a dyke subcritically propagating from a magma chamber indicate that while the general dyke growth characteristics are similar to those with constant creep properties, the subcritical dyke growth velocity is increased by an order of magnitude by considering the temperature dependence of the creep properties. Hence, the subcritical growth duration before the dyke reaches the unstable growth state is significantly shortened.

Key words Mechanics, theory, and modelling; Magma migration and fragmentation; Volcano monitoring.

1 INTRODUCTION
Magma transport through dyke propagation significantly influences the geological evolution of the Earth’s crust and volcanism on the Earth’s surface. Significant progress has been made in understanding the propagation of dykes through the upper mantle and crust (e.g. Spence & Turcotte 1985; Spence et al. 1987; Pollard & Segall 1987; Lister & Kerr 1991; Clemens & Mawer 1992; Rubin, 1995a,b, 1998; Meriaux & Jaupart 1998; Bonafede & Rivalta 1999; Dahm 2000; Ito & Martel 2002; Rivalta & Dahm 2006; Chen et al. 2007; Jin & Johnson 2008).

Most work on dyke propagation has focused on the critical propagation of dykes, that is, the stress intensity factor (SIF) at the dyke tip is equal to or greater than the fracture toughness of the host rock (e.g. Weertman 1971; Spence & Sharp 1985; Spence et al. 1987; Lister 1990, 1991, 1994a,b; Rubin 1995a,b, 1998; Bonafede & Rivalta 1999; Meriaux & Jaupart 1998; Bolchovoy & Lister 1999; Dahm 2000; Menand & Tait 2002; Kuhn & Dahm 2004; Roper & Lister 2005; Rivalta & Dahm 2006; Chen et al. 2007; Taisne & Jaupart 2009; Traversa et al. 2010; Maccaferri et al. 2010). It is expected that small cracks form at the wall of a magma chamber. The SIFs at the tips of those small magma-filled cracks, or dykes, may not reach the fracture toughness and therefore will not propagate critically. However, these small dykes may propagate subcritically due to stress corrosion, viscous damage and other mechanisms.

Anderson & Grew (1977) presented a stress corrosion cracking model as applied to magma fracture initiation. In their model, subcritical growth is due to the weakening of strained atomic bonds at the crack tip by the chemical action of the environment agents. Atkinson (1984) reviewed the progress on subcritical crack growth in geological materials. He discussed common chemical mechanisms responsible for the subcritical growth and presented detailed experimental results for various rocks including granite, marble and limestone. Rubin (1998) examined subcritical crack growth in partially molten rocks from a surface energy point of view. Chen & Jin (2006) investigated subcritical dyke growth from a crustal magma chamber. Using a viscoelastic energy dissipation approach without considering temperature-dependent viscoelastic properties, they obtained the relationship between the subcritical propagation velocity and dyke length, and found that dykes grow at velocities on the order of $10^{-8} - 10^{-6}$ m s⁻¹.

Temperature dependence of the viscoelastic properties of the host rock may play an important role in energy balance during slow, subcritical dyke propagation from a magma chamber. In this paper, we investigate subcritical dyke propagation from a magma chamber using an energy balance approach with viscoelastic energy dissipation. The temperature dependence of the viscoelastic properties and the influence of heat conduction from magma to the host rock around the dyke tip are considered. The relationship between the subcritical growth velocity and dyke length is established.
Numerical results are given to illustrate the effects of magma temperature, magma chamber overpressure and fracture toughness on the subcritical dyke growth velocity.

2 THEORETICAL MODELS

2.1 Magma pressure along the dyke surfaces

Consider a dyke that subcritically propagates vertically from a magma chamber into the host rock. The dyke length is usually much smaller than the size of magma chamber during the subcritical propagation stage. The dyke thus may be considered as a magma filled, plane strain edge crack in a half plane, as shown in Fig. 1, where \( a = a(t) \) denotes the crack length and \( t \) is time. This 2-D dyke propagation model has been adopted in a number of studies, for example, Rubin (1995a,b), Roper & Lister (2005) and Chen & Jin (2006).

Dyke propagation is driven by the net pressure on its surfaces due to magma flow, lithostatic stress and tectonic stress. The fluid pressure \( p \) may be expressed as

\[
p = p_e - (\rho_c g Z + \Delta \sigma),
\]

where \( p_e = p_c(Z) \) is the pressure due to elastic deformation of the host rock, \( Z \) the vertical coordinate, \( g \) the gravitational acceleration, \( \rho_c \) the density of the host rock and \( \Delta \sigma \) the tectonic stress (Rubin 1995a,b; Rope & Lister 2005). Rope & Lister (2005) assumed that \( \Delta \sigma \) could vary linearly in \( Z \)-direction and thus can be absorbed into the lithostatic stress using an effective rock density \( \rho_{\text{eff}} \).

\[
\rho_{\text{eff}} g Z = \rho_c g Z + \Delta \sigma.
\]

The elastic pressure along the dyke surfaces has been derived using the lubrication theory of fluid mechanics as follows (Chen et al. 2007):

\[
p_e = \Delta P + \Delta \rho g(z + a) - 12 \eta V \int_{-a}^{z} \delta^{-2} \, dz,
\]

where \( \Delta P \) is the overpressure in the magma chamber, \( \Delta \rho = \rho_{\text{eff}} - \rho_m \) is the magma density, \( \eta \) is the magma viscosity, \( V \) is the dyke propagation velocity, \( \delta \) is the separation of the two dyke surfaces and \( z = Z - a \). When \( z = -a \) or \( Z = 0 \), \( p_e = \Delta P \).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{dike.png}
\caption{A dyke in a semi-infinite medium (\( a \): dyke length, \( \delta \): dyke thickness, \( p \): dyke surface pressure).}
\end{figure}

2.2 Temperature field and viscoelastic constitutive equations with temperature-dependent creep properties

Subcritical dyke growth in the crust takes place generally near a magma chamber where the material properties of the host rock are greatly influenced by the elevated temperatures. The temperature distribution around a dyke satisfies the following heat conduction equation:

\[
\frac{\partial^2 T}{\partial Z^2} + \frac{\partial^2 T}{\partial X^2} = 0,
\]

where \( T \) is the temperature. Here the transient effect of the temperature field is neglected as the subcritical dyke growth velocities are very low (Chen et al. 2006). The temperatures along the magma chamber (\( Z = 0 \)) and dyke (\( X = 0, 0 \leq Z \leq a \)) are assumed as the melting temperature of the magma, \( T_m \). By solving eq. (4) we can get the temperature field as follows:

\[
T(X, Z) = T_m - \frac{T_m - T_i}{h} Z - \frac{1}{2\pi} \int_{0}^{a} \ln \left( \frac{X^2 + (Z' + Z)^2}{X^2 + (Z' - Z)^2} \right) f(Z') \, dZ',
\]

where \( h \) is the depth of dyke base, \( T_i \) is the temperature on the Earth’s surface and \( f(Z') \) is determined from the integral equation (A7) in the Appendix in which the detailed derivation of eq. (5) is given.

The relationship between creep strain rate and differential stress may be modelled by the following power law for crustal rock under high-temperature creep conditions (Kirby et al. 1987):

\[
\dot{\varepsilon} = A(\sigma_1 - \sigma_2)^n \exp \left\{ \frac{-Q}{RT} \right\},
\]

where \( \dot{\varepsilon} \) is creep strain, \( \sigma_i \) (\( i = 1, 3 \)) are stresses, \( T \) is the absolute temperature, \( A, n \) and \( Q \) are constants related to rock properties and \( R \) is the universal gas constant. Eq. (6) is generally a non-linear relation between the strain rate and the stress. For simplicity, we use a special case of \( n = 1 \) as follows:

\[
\dot{\varepsilon} = A'(\sigma_1 - \sigma_2) \exp \left\{ \frac{-Q}{RT} \right\},
\]

where \( A' \) is a material constant. Eq. (7) will be used in Section 3 for calculating the viscous dissipative energy.

2.3 SIF calculation

For the sake of simplicity, we neglect the effect of the temperature dependence of elastic modulus in calculating the SIF at the dyke tip. Eq. (3) indicates that the boundary condition on the dyke (or crack) surfaces is non-linear and related to the crack opening. Using a combined perturbation/integral equation approach, Chen et al. (2007) obtained the expression of the SIF at the dyke tip as follows:

\[
K_1 = -\frac{\Delta \rho g a}{2} \sqrt{\pi a} \left[ \tilde{\psi}_0(1) + e \tilde{\psi}_1(1) + e^2 \tilde{\psi}_2(1) + e^3 \tilde{\psi}_3(1) \right],
\]

where \( \tilde{\psi}_i(r)(i = 0, 1, 2, 3) \) are dimensionless functions given by

\[
\tilde{\psi}_i(r) = \sqrt{1 - r^2 \tilde{\psi}(r)}, \quad i = 0, 1, 2, 3,
\]

and the dimensionless functions \( \tilde{\psi}_i(r)(i = 0, 1, 2, 3) \) satisfy the

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following integral equations:

\[
\begin{align*}
\int_{-1}^{1} \left[ \frac{1}{s-r} + K(r,s) \right] \psi_0(s) \, ds &= -\frac{2\pi}{\Delta \rho g a} \Delta P - \pi (1 + r), \\
\int_{-1}^{1} \left[ \frac{1}{s-r} + K(r,s) \right] \psi_1(s) \, ds &= \pi \left[ \frac{E}{(1 - v^2)\Delta \rho g a} \right]^2 \left( \frac{H}{\delta_0} \right)^2 \int_{-1}^{1} \psi_0(s) \, ds, \\
\int_{-1}^{1} \left[ \frac{1}{s-r} + K(r,s) \right] \psi_2(s) \, ds &= \pi \left[ \frac{E}{(1 - v^2)\Delta \rho g a} \right]^2 \times \int_{-1}^{1} \left[ -2 \Delta \frac{\partial (r/H \delta_0)}{\partial s} \right]^2 \psi_0(s) \, ds, \\
\int_{-1}^{1} \left[ \frac{1}{s-r} + K(r,s) \right] \psi_3(s) \, ds &= \pi \left[ \frac{E}{(1 - v^2)\Delta \rho g a} \right]^2 \times \int_{-1}^{1} \left( \frac{H}{\delta_0} \right)^2 \left( \frac{3 \delta_0^2 - 2 \delta_2}{\delta_0} \right) \psi_0(s) \, ds.
\end{align*}
\] (10)

respectively, and \( \varepsilon \) is a non-dimensional perturbation parameter given by

\[
\varepsilon = \frac{12\eta V}{H^2} \frac{1}{\Delta \rho g}.
\] (11)

In eqs (9)–(11), \( r = 2Z/a - 1 \), \( Z \) is the vertical coordinate as shown in Fig. 1, \( K(r,s) \) is a known kernel, \( H \) is a length parameter having an order of the dyke base thickness, and \( \delta_i(r) (i = 0, 1, 2, 3) \) are related to crack opening given by

\[
\delta_i(r) = -a \int_{r}^{1} \psi_i(s) \, ds, \quad i = 0, 1, 2, 3.
\] (12)

It is noted that \( \Delta P \) is assumed as a constant as in most dyke propagation studies while the buoyancy pressure \( \Delta \rho g a \) varies with dyke length. The integral equations in eq. (10) are solved numerically.

2.4 Analysis of subcritical crack propagation in a viscoelastic solid with temperature-dependent creep properties

Under isothermal conditions, the appropriate statement of the global conservation of energy for crack extension in viscoelastic media is (Christensen 1982)

\[
dU \frac{dr}{da} + dD_p \frac{dr}{da} + dS_e \frac{dr}{da} = 0,
\] (13)

where \( U \) is the elastic strain energy per unit thickness, \( D_p \) is the viscous dissipation energy, \( S_e \) is the surface energy and \( t \) is time. For the growth of an edge crack of length \( a \), eq. (13) may also be written as

\[
dU \frac{dr}{da} + dD_p \frac{dr}{da} + dS_e \frac{dr}{da} = 0,
\] (14)

where \( da = V \, dr \) with \( V \) being the growth velocity.

To study subcritical dyke propagation, we assume that an initial short dyke of length \( a_0 \) is first produced around a magma chamber (Sleep 1988; Fowler 1990). The dyke then subcritically grows in a step-by-step manner, that is, the dyke grows to a length \( a \) through a time period \( T_g(a_0) \) and then stops. After another time period \( T_g(a) \), the dyke continues to grow to a new length and stops again. Furthermore, there exists a critical length \( a_c \) at which the SIF at the dyke tip reaches the critical SIF. \( T_g(a) \) is equal to zero when \( a \) is larger than \( a_c \), which means the termination of subcritical growth stage and the dyke will grow unstably until failure occurs. In general, \( T_g(a) \) is the time period from the stop of the last dyke extension up to the start of the next dyke extension. \( T_g(a) \) may be understood as the time for unit subcritical dyke extension. The analysis process mentioned earlier is a numerical strategy to solve our problem.

For the numerical process of subcritical crack extension described earlier, the energy terms \( dU/da, dD_p/da \) and \( dS_e/da \) in eq. (14) have been obtained in Chen (2003), Chen & Bai (2006) and Chen & Jin (2006) for a mode I crack/dyke for plane strain. We summarize these descriptions as follows:

For mode I crack extension problems for plane strain, the strain energy change rate relates to the SIF by (Knott 1973)

\[
dU \frac{da}{dr} = -\frac{2(1 - v^2)}{E} K_i^2,
\] (15)
where $E$ is Young’s modulus, $\nu$ is Poisson ratio and $K_I$ is the SIF. Here, we neglect the dependence of Young’s modulus on the temperature. The stresses around the crack tip do not vary with time during a small crack growth step in accordance with the correspondence principle (Christensen 1982). The crack tip stress field for a mode I crack is

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta),$$

where $r$ and $\theta$ are the polar coordinates centred at the crack tip at the end of the step, $\sigma_{ij}$ is stress tensor and $f_{ij}(\theta)$ are the angular distributions of stresses.

The viscoelastic constitutive equations for creep may be written as follows:

$$e_{ij} = s_{ij} G(t), \quad e_v = p K(t),$$

$$e_{ij} = e_{ij} - e_v, \quad e_v = \epsilon_{ii}/3, \quad s_{ij} = \sigma_{ij} - p_v \delta, \quad p_v = \sigma_{ii}/3,$$

where $G(t)$ and $K(t)$ are the deviatoric and volumetric compliances, respectively, $s_{ij}$ is the deviatoric stress tensor, $p_v$ is the hydrostatic stress, $e_{ij}$ is the deviatoric strain tensor, $e_v$ is the volumetric strain and $e_{ij}$ is the strain tensor.

In a crack extension/stop step, the dissipative energy density $Q_e$ at a point of the medium may be calculated as

$$Q_e = \int_0^{T_g} \sigma_{ij} \dot{e}_{ij} \, d\zeta,$$

where $T_g$ is the time period from the stop of the last dyke extension up to the start of the next dyke extension and a dot over $\epsilon_{ij}$ denotes strain rate.

It is assumed that the dissipation of energy is due to the stress singularity at the crack tip. Combining eqs. (16) and (18), the total dissipative energy $dD_p$ per unit thickness in the step can be calculated by integrating $Q_e$ over a crack tip area surrounded by a small circle around the crack tip as follows:

$$dD_p = -\int_0^{T_g} \int_{\pi}^{0} Q_e r \, d\theta \, dr = -\pi K_I^2 [B(T_g) - B(0)] \, da,$$

where $B(T_g)$ is defined by

$$B(T_g) = \left( \frac{25}{24} - \frac{4}{3} \nu \right) G(T_g) + \frac{4}{3} (1 + \nu) K(T_g).$$

In eq. (20), $G(T_g)$ and $K(T_g)$ are the deviatoric and volumetric compliances of the viscoelastic solid.

The surface energy rate is given by

$$\frac{dSe}{da} = 2\gamma,$$

where $\gamma$ represents the surface energy per unit area and is related to the fracture toughness $K_{IC}$ by (Jaeger and Cook, 1976)

$$\gamma = \frac{(1 - \nu^2)}{E} K_{IC}^2.$$

Substituting eqs. (15), (19) and (21) into eq. (14), we have

$$B(T_g) = \frac{4}{\pi K_I^2} \left[ \left( K_{IC}^2 - K_I^2 \right) \times (1 - \nu^2) \right] + B(0).$$

Figure 3. Dyke subcritical growth velocity versus the magma temperature for various dyke lengths ($\Delta P = 3.0$ Mpa, $K_{IC} = 52$ MPa m$^{3/2}$).
The above eq. (23) must be satisfied by $T_g(a)$.

Next we evaluate the function $B(T_g)$ for the following temperature-dependent creep functions (Miannay 2001):

$$G(T_g) = A'T_g \exp\left(-\frac{Q}{RT}\right),$$

$$K(T_g) = 0.$$  \hspace{1cm}(24)

Eq. (24) may be obtained from eq. (7) for creep process with the rock constants determined by creep experiments. From the numerical result of the temperature distribution around a dyke (as shown in Fig. 2), the temperature in eq. (24) may be taken as the magma temperature. Substituting eqs (20) and (24) into (23) and solving for $T_g = T_g(a)$, we have

$$T_g(a) = \frac{288 \times (1 - \nu^2)}{A' \pi E \times (75 - 96\nu)} \left(\frac{K_{IC}}{K_f}\right)^2 - 1 \exp\left(\frac{Q}{RT}\right).$$  \hspace{1cm}(25)

In accordance with the definition of $T_g(a)$ (time period for unit subcritical dyke extension), the relation between $T_g(a)$ and the subcritical dyke growth velocity, $V$, can be established as

$$V = \frac{1}{T_g(a)}. \hspace{1cm}(26)$$

3 RESULTS AND DISCUSSION

In our calculations, we use the following typical properties for the host rock and magma (Rubin 1995a,b; Kirby et al. 1987): $E = 50$ GPa, $\nu = 0.25$, $\Delta \rho = 300$ kg m$^{-3}$, $\eta = 50$ Pa-s, $g = 9.8$ m s$^{-2}$.

Thermally activated Newtonian creep (eq. 7) of crustal rocks corresponds to a viscosity decreasing with temperature. The temperature-dependent viscosity of rock $\eta_{rock}$ may be expressed as follows:

$$\eta_{rock} = \frac{1}{A'} \exp\left(\frac{Q}{RT}\right).$$  \hspace{1cm}(27)

Using $A' = 0.0001$ MPa$^{-1}$ s$^{-1}$ and $Q = 137$ kJ mol$^{-1}$ (Turcotte & Schubert 2002) yields $\eta_{rock} = 4.049 \times 10^{14}, 4.175 \times 10^{15}, 1.569 \times 10^{16}$ and $4.272 \times 10^{20}$ Pa-s at temperatures of 1280 °C, 1000 °C, 600 °C and 400 °C, respectively. We perform a parametric study on the effects of physical parameters (fracture toughness, magma temperature and magma chamber pressure) on the subcritical dyke growth velocity. The SIFs are calculated using eq. (8).

In this work, we only consider steady-state temperature distribution by thermal conduction process based on the fact that dyke subcritical growth is very slow. Fig. 2 shows the isothermal curves (constant temperature) calculated from eq. (5). The magma temperature, dyke length, dyke base depth and surface temperature are assumed as $T_m = 1020$ °C, $a = 500$ m, $h = 20000$ m and $T_s = 0$, respectively. It can be seen that the temperature near the dyke is close to the magma temperature and decreases with increasing distance from the dyke. The numerical results suggest that the magma temperature may be used in eq. (24) for determining the temperature-dependent creep properties used for crack tip energy balance calculations.

The dyke subcritical growth velocity $V$ versus magma temperature behaviour is shown in Figs 3–5. The effect of initial dyke length on the subcritical growth behaviour is examined in Fig. 3 where the magma chamber overpressure is $\Delta P = 3.0$ MPa and

![Figure 4. Dyke subcritical growth velocity versus the magma temperature for various dyke lengths ($\Delta P = 3.0$ Mpa, $K_{IC} = 100$ MPa m$^{1/2}$).](image)
fracture toughness \( K_{IC} = 52 \text{ MPa m}^{1/2} \). It is seen from Fig. 3 that at a given dyke length, \( V \) increases with an increase in \( T_m \). The subcritical velocity is on the order of \( 10^{-7} - 10^{-4} \text{ m s}^{-1} \), much smaller than the unstable propagation velocities in the range of \( 0.01-10 \text{ m s}^{-1} \) (Rubin 1995a,b). The subcritical growth velocity increases rapidly when the dyke is approaching the critical length (71 m) at which the SIF reaches the fracture toughness. The results indicate significant effects of temperature-dependent creep properties of the host rock on the subcritical growth velocity. For example, at a dyke length of 70 m, \( V \) increases to \( 5 \times 10^{-4} \text{ m s}^{-1} \) when \( T_m \) is 1250 °C. Clearly, high temperature-induced thermal dissipation in the host rock results in earlier unstable dyke propagation than that without considering the effects of temperature-dependent creep properties of the host rock.

Fig. 4 shows the subcritical growth velocity versus magma temperature for various dyke length at an increased fracture toughness of \( K_{IC} = 100 \text{ MPa m}^{1/2} \). This fracture toughness value may be near the high end of the range for basaltic oceanic crust as the toughness of crustal rock generally increases with increasing confining pressure (Jin & Johnson 2008). The magma chamber overpressure is still \( \Delta P = 3 \text{ MPa} \). Similar subcritical dyke growth behaviour can be observed. However, the dyke growth velocity is significantly lower compared with that for the case of \( K_{IC} = 52 \text{ MPa m}^{1/2} \) shown in Fig. 3 under the same magma temperature and dyke length. We note that the critical dyke length is now 220 m and longer than that for \( K_{IC} = 52 \text{ MPa m}^{1/2} \).

The effect of magma chamber overpressure \( \Delta P \) on the subcritical dyke growth behaviour is examined in Fig. 5. The initial dyke length is now fixed at 85 m and the fracture toughness is \( K_{IC} = 100 \text{ MPa m}^{1/2} \). It is seen that the subcritical growth velocity increases with increasing magma chamber pressure for a given magma temperature. Hence, higher \( \Delta P \) will result in early unstable dyke propagation.

Fig. 6 shows the dyke growth duration versus dyke length under the conditions of \( \Delta P = 3 \text{ MPa} \), \( K_{IC} = 52 \text{ MPa m}^{1/2} \) and \( T_m = 1000 \degree \text{C}, 1100 \degree \text{C} \) and 1280 °C. When the temperature of magma is 1000 °C, dyke subcritical growth is very slow and it needs about 1.48 yr for the dyke to reach 70 m, which is almost equal to the critical dyke length 71 m. When the temperature of magma is 1100 °C, dyke subcritical growth becomes faster and it needs about 0.58 yr for the dyke to reach 70 m. When the temperature of magma is 1280 °C, dyke subcritical growth speeds up and it only needs about 0.14 yr for the dyke to reach 70 m. Dyke growth becomes unstable when the dyke approaches 71 m.

According to Turcott & Schubert (2002), the solidification time \( t_s \) of a dyke may be calculated as follows:

\[
t_s = \frac{b^2}{4 \kappa \lambda_2^2},
\]

where \( b \) is the dyke width, \( \kappa \) is the thermal diffusivity and \( \lambda_2 \) is a parameter depending on the difference between the magma temperature and the host rock temperature. Taking a dyke width \( b = 1 \text{ m} \) (which is approximately equal to the opening at the dyke base in our calculations), thermal diffusivity \( \kappa = 0.5 \text{ mm}^2 \text{ s}^{-1} \) and \( \lambda_2 = 0.2 \) (which corresponds a temperature difference \( T_m - T_0 = 100 \degree \text{C} \), see Fig. 2), we get \( t_s = 144.7 \text{ d}, \) or 0.406 yr. The calculation above is based on a stationary dyke and hence is conservative. The actual solidification time may be longer for a growing dyke as new

\[\text{Figure 5. Dyke subcritical growth velocity versus the magma temperature for various magma chamber pressures (} b = 85 \text{ m, } K_{IC} = 100 \text{ MPa m}^{1/2}).\]
magasms flow from the magma chamber into the dyke during propagation. Hence, the dyke may solidify during its subcritical growth when the magma temperatures are 1000 °C and 1100 °C. When the magma temperature is 1280 °C, the dyke can reach the critical state of unstable propagation before complete solidification occurs. In our analysis, the elastic modulus $E$ is assumed to be constant and does not depend on the temperature. Based on the experimental results of Liu et al. (2000), the elastic modulus of granite reduces to about 77 per cent of its room temperature value at a temperature of 1280 °C. Assuming a temperature-independent elastic modulus will likely cause a relative error on the order of 20 per cent in the calculated subcritical growth velocity. Consideration of the temperature-dependent creep properties, however, will increase the subcritical velocity by an order of magnitude as indicated in Figs 3–5 as compared with our previous results with temperature-independent creep properties. Hence, temperature dependence of creep properties is more dominant and the effect of temperature dependence of elastic modulus on the subcritical dyke propagation may be neglected.

4. CONCLUSION

In this paper, we obtain the temperature field around a dyke subcritically growing from a magma chamber and investigate the effects of temperature-dependent creep properties of the host rock on the dyke subcritical growth behaviour. An energy balance approach is used which considers energy dissipations due to viscoelasticity including temperature effect. The numerical results indicate that the energy dissipation due to higher temperature of the host rock at the dyke tip results in higher subcritical propagation velocity compared with that without considering the effects of temperature-dependent creep properties of the host rock. The subcritical propagation velocity increases with an increase in the magma temperature. We also find that the fracture toughness $K_{IC}$ and the magma chamber overpressure $\Delta P$ have profound effects on the subcritical dyke propagation velocity. Subcritical growth of dyke is a generally slow process with the velocity in the range of $10^{-7}–10^{-4}$ m s$^{-1}$ before the dyke length reaches its critical length. A subcritically growing dyke may or may not reach the critical state of unstable propagation depending on the magma solidification rate and the temperature in the host rock around the magma chamber. Inclusion of the effects of temperature-dependent creep properties of the host rock will increase the chance for the dyke to reach the unstable state.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (40874046). The authors would also like to thank the editor and two reviewers for helpful discussions and comments.

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**APPENDIX: DETAILED DERIVATION OF TEMPERATURE FIELD**

The boundary conditions of the steady-state thermal conduction problem for a dyke at a magma chamber (modelled as a magma filled edge crack in a half-space) are

\[ T = T_m \quad Z = 0, \]
\[ T = T_s \quad Z = h, \]
\[ T = T_m \quad X = 0, \quad 0 \leq Z \leq a \]
\[ \frac{\partial T}{\partial X} = 0, \quad X > 0, \quad Z > a. \quad (A1) \]

The temperature in the host rock can be expressed as

\[ T(X, Z) = T_{13}(Z) + T_{12}(X, Z), \quad (A2) \]

where \( T_{13}(Z) \) is the temperature without considering the dyke and is given as

\[ T_{13}(Z) = T_m - \frac{T_m - T_s}{h} Z. \quad (A3) \]

The basic equation and the boundary conditions for \( T_{12}(Z) \) are

\[ \frac{\partial^2 T_{12}}{\partial Z^2} + \frac{\partial^2 T_{12}}{\partial X^2} = 0, \]
\[ T_{12} = 0 \quad Z = 0, \quad X \geq 0, \]
\[ T_{12} = \frac{T_m - T_s}{h} Z, \quad X = 0, \quad 0 < Z < a, \]
\[ \frac{\partial T_{12}}{\partial X} = 0 \quad X = 0, \quad Z > a. \quad (A4) \]
The solution of the above problem can be obtained using Fourier transform as follows;

\[ T_{(2)}(X, Z) = \int_{0}^{\infty} \alpha(\xi)e^{-\xi X} \sin(\xi Z) \, d\xi, \]

\[ a(\xi) = -\frac{2}{\pi\xi} \int_{0}^{a} f(Z) \sin(\xi Z') \, dZ', \]

where \( f(Z) \) is defined by

\[ \frac{\partial T_{(2)}}{\partial X} = f(Z) \quad X = 0, \quad 0 < Z < a, \]  

(A5)

and satisfies the following integral equation

\[ -\frac{1}{\pi} \int_{0}^{a} f(Z') \ln \left| \frac{Z' + Z}{Z' - Z} \right| \, dZ' = \frac{T_{m} - T_{c}}{h} Z \quad 0 \leq Z \leq a. \]  

(A7)

The final expression for \( T_{(2)}(Z) \) has the following form:

\[ T_{(2)}(X, Z) = -\frac{1}{2\pi} \int_{0}^{a} \ln \left( \frac{X^{2} + (Z' + Z)^{2}}{X^{2} + (Z' - Z)^{2}} \right) f(Z') \, dZ'. \]  

(A8)