BRDF Model Inversion of Multiangular Remote Sensing: Ill-posedness and the Interior Point Solution Method

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Abstract: Evaluation of the land surface albedos by employing the bidirectional reflectance distribution function (BRDF) models is one of the important problems in remote sensing. As is known, the retrieval process is an inverse problem. In Proposition 3 of [Verstraete et al., 1996], the authors consider that the number of independent observations should be greater than the number of the unknown parameters to describe the physical model as an overdetermined system, then the inverse process can be solved. However as Li et al (1998) pointed out that such a requirement can be hardly satisfied even in the coming EOS era, the inversion procedure is always underdetermined in some sense. Therefore, in order to solve the BRDF inversion problem, some new technique must be developed. Generally speaking, the inverse problems are ill-posed. Therefore, some regularization technique should be applied to suppress the ill-posedness. One kind of way to alleviate the ill-posedness is incorporating with some apriori knowledge which has been developed in [Li et al., 2001]. This is actually a constrained least squares error (LSE) method since the apriori knowledge can be considered as some kind of constraints to the solution. Another kind of way is by numerical truncated singular value decomposition by employing the hotspot remote sensing data [Wang et al., 2006]. In this paper, we consider a new solution method, i.e., the l^1 norm solution method, which iteratively solves the kernel-driven bidirectional reflectance distribution function (BRDF) models for retrieval of land surface albedos. This method, is based on searching for an interior point solution for the problem in the feasible solution set. This method can always find a set of suitable BRDF coefficients for poor sampled data. Numerical performance is given for the widely used 18 data sets among the 73 data sets [Li et al., 2001].

1. Introduction

The real physical system that couples the atmosphere and the land surface is very complicated and should be continuous, sometimes it requires a comprehensive parameters to describe such a system, so any practical physical model can only be approximated by a model which includes only a limited number of the most important parameters that capture the major variation of the real system. Generally speaking, a discrete forward model to describe such a system is in the form

$$\mathbf{D} = h(\mathbf{X}, \mathbf{S}) + \mathbf{n}, \tag{1}$$

where D is a vector in $\mathbb{R}^M$, which is an M dimensional measurement space with M values corresponding to M different measurement conditions, $\mathbf{n} \in \mathbb{R}^M$ is the vector of random noise with same vector length M, $\mathbf{X}$ is a vector of controllable measurement conditions such as wave band, viewing direction, time, Sun position, polarization, and the forth, $\mathbf{S}$ is a vector of state parameters of the system approximation, h is a function which relates $\mathbf{X}$ with $\mathbf{S}$, which is generally nonlinear and continuous. Assume that there are m undetermined parameters need to be recovered. If more observations can be collected than the existing parameters in the model, i.e., $M > m$ [Verstraete et al., 1996], the system (1) is over-determined. In this situation, the traditional solution does not exist. We must define its solution in some other meaning, for example, the least squares error (LSE) solution. However as Li [Li et al 1998] pointed out that, "for physical models with about ten parameters (single band), it is questionable whether remote sensing inversion can be an over-determined one in the foreseeable future." Therefore, the inversion problems in geoscience seem to be always underdetermined in some sense.

The bidirectional reflectance distribution function (BRDF) models are a main class of models which can be inverted to estimate structural parameters and spectral component signatures of Earth surface cover type [Strahler et al., 1999, Wanner et al., 1995, Roujean et al., 1992]. As is well-known, the state-of-the-art of BRDF is the use of the linear kernel-driven models, mathematically described as the linear combination of the isotropic kernel, volume scattering kernel and geometric optics kernel. The computational stability is characterized by the algebraic operator spectrum of the kernel-matrix and the observation errors. Therefore, the retrieval of the model coefficients is of great importance for computation of the land surface albedos. In [Pokrovsky et al., 2002], the authors have utilized the QR decomposition for inversion of the BRDF model. Later on, in [Wang et al., 2006], the authors consider the singular value decomposition and propose a regularized version of the method. However, all of the methods are based on the direct solution of the linear system by avoiding the direct inversion of the finite rank matrix. This paper consider the iterative solution methods for retrieval land surface parameters. This method help us to find a suitable solution in the feasible set for poor sampled data.

2. Kernel-driven BRDF Model

As advances in the field of multiangular remote sensing, it is increasingly probable that BRDF models can be inverted to estimate the important biological or climatological parameters of the earth surface such as leaf area index and albedo [Strahler et al., 1994]. A linear kernel driven BRDF model is usually described in the following form [Roujean et al., 1992]:

$$f_{iso} + k_{vol}(t_i, t_v, \phi) f_{vol} + k_{geo}(t_i, t_v, \phi) f_{geo} = r(t_i, t_v, \phi), \quad (2)$$

where r is the bidirectional reflectance; k_{vol} and k_{geo} are so-called kernels, i.e., known functions of illumination and viewing geometry which describe volume and geometric scattering respectively; t_i is the zenith angle of the solar direction, t_v is the zenith angle of the view direction; ϕ is the relative azimuth of Sun and view direction; f_{iso}, f_{vol} and f_{geo} are three unknown coefficients to be adjusted to fit observations. In [Wang et al., 2006], the model (2) is considered as the discretized linear operator equations

$$K\mathbf{f} = \mathbf{r}. \quad (3)$$

Generally speaking, the BRDF model should includes different kernels of many types. However, it was proved that the combination of RossThick (k_{vol}) and LiSparse (k_{geo}) kernels had the best overall ability to fit BRDF measurements and to extrapolate for BRDF and albedo [Hu et al., 1997; Wanner et al., 1995]. A suitable expression for k_{vol} was derived by Roujean [Roujean et al., 1992], i.e., the RossThick kernel; A suitable expression for k_{geo} was derived by [Li et al., 2000], i.e., the LiTransit kernel, we refer to these articles for details.

3. Parameter Retrieval Method: Interior Point Method for Poor Sampled Data

It is clear that when the number of looks are insufficient or the location is poor, the physical problem (1), so as (2) is ill-posed. The ill-posedness occurs not only for the instability driven by small algebraic characteristic spectrum but also for choosing a suitable solution from the solution set consists of infinite solutions. It deserves attention that the ill-posedness is the intrinsic feature of the inverse problems. Unless some additional information/knowledge such as monotonicity, smoothness, boundedness or the error bound of the raw data are imposed, the difficulty is hardly to be solved. As is pointed out in [Lanczos, 1961] that, a lack of information can not be remedied by any mathematical trickery. However, we can retrieve (most of) the information of the original problem by improvement of the solvability by extension of the solution space.

Generally speaking, the kernel-driven BRDF model is semiempirical, the retrieved parameters f are mostly considered as a kind of weight function though it is a function of LAI and other related geometric parameters. Therefore, f is not necessarily positive. However, since it is a weight function, an appropriate arrangement of the components of f can yield the same results. That is to say, f can be "made" to be positive. The remaining problem is to develop some proper method. Our new meaning to the solution is related to the l^1 norm problem

$$\begin{aligned} &\min_{\mathbf{f}} \ \|\mathbf{f}\|_{l^1}, \\ &\text{s.t. } K\mathbf{f} = \mathbf{r}, \ \mathbf{f} \geq 0. \end{aligned} \quad (4)$$

The l^1 norm solution method is seeking for a feasible solution within the feasible set $S = \{\mathbf{f} : K\mathbf{f} = \mathbf{r}, \ \mathbf{f} \geq 0\}$. So it is actually searching for an interior point within the feasible set S, hence is called the interior point method. The dual standard form of (4) is in the form

$$\begin{aligned} &\max \ \mathbf{r}^T \mathbf{g}, \\ &\text{s.t. } s = e - K^T \mathbf{g} \geq 0. \end{aligned} \quad (5)$$

Therefore, the optimality conditions for $(\mathbf{f}, \mathbf{g}, s)$ to be a primal-dual solution triplet are that

$$K\mathbf{f} = \mathbf{r}, \ K^T\mathbf{g} + s = e, \ \tilde{S}\tilde{F}e = 0, \mathbf{f} \geq 0, \ s \geq 0, \quad (6)$$

where $\tilde{S} = \operatorname{diag}(s_1, s_2, \cdots, s_N)$, $\tilde{F} = \operatorname{diag}(\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_N)$. The notation $\operatorname{diag}(\cdot)$ denotes the diagonal matrix whose only nonzero components are the main diagonal line.

The interior point method generates iterates $\{\mathbf{f}_k, \mathbf{g}_k, s_k\}$ such that $\mathbf{f}_k > 0$ and $s_k > 0$. As the iteration index k approaches infinity, the equality-constraint violations $\|\mathbf{r} - K\mathbf{f}\|$ and $\|K^T\mathbf{g}_k + s_k - e\|$ and the duality gap $\mathbf{f}_k^T s_k$ are driven to zero, yielding a limiting point that solves the primal and dual linear problems. The primal-dual solution is obtained by a variant of Newton's method applied to the system of equations formed by the optimality conditions (6).

4. Numerical Performance

In this section, we give some numerical results to show that the interior point solution method is suitable for retrieval parameters for poor sampled data. Assume that there are M different measurement kernel driven models, then (2) can be rewritten in the matrix-vector form

$$\mathcal{K}\overrightarrow{X} = \overrightarrow{Y}, \quad (7)$$

where $\mathcal{K} \in \mathbb{R}^{M\times 3}$, $\overrightarrow{X} \in \mathbb{R}^3$, $\overrightarrow{Y} \in \mathbb{R}^M$. In practice, the vector $\overrightarrow{Y}$ should also include different kind of noise. For simplicity, we assume that the noise is additive, i.e., $\mathcal{K}\overrightarrow{X} = \overrightarrow{Y}_\delta := \overrightarrow{Y} + \delta\overrightarrow{n}$, where δ is the noise level in (0,1). We also assume that $\|\overrightarrow{Y}_\delta - \overrightarrow{Y}\| \leq \tau\delta < \|\overrightarrow{Y}_\delta\|$, where $\tau > 1$. This assumption indicates that the signal-to-noise ratio (SNR) should be greater than 1, otherwise we consider the observations (BRDF) is not believable. It is clear that (7) is an underdetermined system if $M \leq 2$ and an overdetermined system if $M > 3$.

In our test, the insufficient look is chosen as the hotspot data, details and explanation are given in [Wang et al., 2006]. We use the widely used 73 data sets [Li et al., 2001]. Among the 73 sets of BRDF measurements, only 18 sets of field-measured BRDF data with detailed information about the experiment were chosen, including biophysical and instrumental information. For the summary of the basic properties of the data, we refer to [Wang et al., 2006]. These data sets cover a large variety of vegetative cover types, and are fairly well representative of the natural and cultivated vegetation.

We regard the retrieval results from multiangular views as "true" values, and compare the interior point solutions to the "true" values. We only list the retrieval results of Kimes's data in visible and near infrared bands. From Table 1-Table 2, we find that the albedos retrieved from one look, two looks and multiangular looks by our algorithm coincide with each other satisfactorily (i.e., in our trust region) though there are obvious deviation among the corresponding values. For other data, such as Ranson' and Parabola's data, the retrievals are also reasonable.

Table 1. Computational values of the WSAs of Kimes' data in Vis band

	Single Look	Two Looks	Multiangular
corn	0.07220191168879	0.11988208939852	0.077371794
hardwood	0.03900181988865	0.02035687953748	0.036017748
irrwheat	0.07330182359673	0.05811263968740	0.066419290
lawn	0.04790188339507	0.06703131273902	0.057035696
orchgrass	0.04910189229549	0.10968810772315	0.078334436
soy	0.00772219426508	0.04000072690462	0.037576732

Table 2. Computational values of the WSAs of Kimes' data in Nir band

	Single Look	Two Looks	Multiangular
corn	0.28150185720982	0.27411364595928	0.288654970
hardwood	0.32000182018128	0.19251564731464	0.369430037
irrwheat	0.62100182359988	0.52687758932043	0.513398848
lawn	0.40270185630598	0.51468506078909	0.412934981
orchgrass	0.25370185454194	0.39972500975936	0.296322714
soy	0.06853878697192	0.59945163336588	0.515229716

5. Conclusion

This paper has considered the interior point iterative method for the solution of the inverse problems in land surface parameter retrieval. It is clear that l^1 norm solution is a special case of l^p norm solution for $0 < p < \infty$. It is expected that the l^p norm solution is more applicable in applications.

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